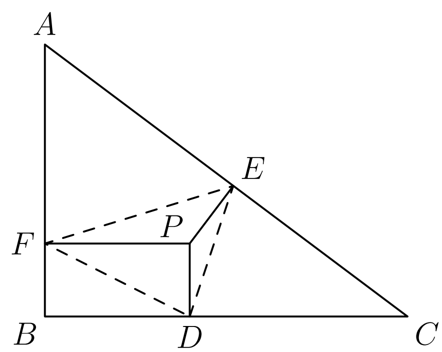
San Diego Math League High School Division, Round 2b
January 21, 2012

1. The function f is defined by $f(x) = x^2 + 3x$. Find the product of all solutions of the equation $f(2x-1) = 6$.

2. The 120 permutations of the word BORIS are arranged in alphabetical order, from BIORS to SROIB. What is the 60^{th} word in this list?

3. The 42 points $P_1, P_2, \ldots, P_{42}$ lie on a straight line, in that order, so that the distance between P_n and P_{n+1} is $\frac{1}{n}$ for all $1 \leq n \leq 41$. What is the sum of the distances between every pair of these points? (Each pair of points is counted only once.)

4. In triangle ABC, $AB = 3$, $AC = 5$, and $BC = 4$. Let P be a point inside triangle ABC, and let D, E, and F be the projections of P onto sides BC, AC, and AB, respectively. If $PD : PE : PF = 1 : 1 : 2$, then find the area of triangle DEF. (Express your answer as a reduced fraction.)



5. What is the greatest number of regions into which four planes can divide three-dimensional space?

6. A positive integer is equal to the sum of the squares of its four smallest positive divisors. What is the largest prime that divides this positive integer?

7. Let x and y be nonnegative real numbers such that $x + y = 1$. Find the maximum value of $x^4y + xy^4$.

8. The distinct positive integers a and b have the property that

$$\frac{a+b}{2}, \quad \sqrt{ab}, \quad \frac{2}{\frac{1}{a}+\frac{1}{b}}$$

are all positive integers. Find the smallest possible value of $|a-b|$.